Open Quantum Systems in Noninertial Frames

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Abstract

We study the effects of decoherence on the entanglement generated by Unruh effect in noninertial frames by using bit flip, phase damping and depolarizing channels. It is shown that decoherence strongly influences the initial state entanglement. The entanglement sudden death can happens irrespective of the acceleration of the noninertial frame under the action of phase flip and phase damping channels. It is investigated that an early sudden death happens for large acceleration under the depolarizing environment. Moreover, the entanglement increases for a highly decohered phase flip channel.
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1 Introduction

Entanglement is one of the potential sources of quantum theory. It is the key concept and major resource for quantum communication and computation [1]. In the last few years, enormous efforts has been made to investigate various aspects of quantum entanglement and its benefits in a number of setups, such as teleportation of unknown states [2] , quantum key distribution [3], quantum cryptography [4] and quantum computation [5, 6]. Recently, the study of quantum entanglement of various fields has been extended to the relativistic setup [7, 8, 9, 10, 11, 12] and interesting results about the behavior of entanglement have been obtained. The study of entanglement in the relativistic framework is important not only from quantum information perspective but also to understand deeply the black hole thermodynamics [13, 14] and the black hole information paradox [15, 16].

The earlier investigations on quantum entanglement in the relativistic framework is mainly focused by considering isolated quantum systems. In fact, no quantum system can be completely isolated from its environment and may results in a non-unitary dynamics of the system. Therefore, it is important to

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study the effect of environment on the entanglement in an initial state of a quantum system during its evolution. The interaction between an environment and a quantum system leads to the phenomenon of decoherence and it gives rise to an irreversible transfer of information from the system to the environment [17, 18, 19].

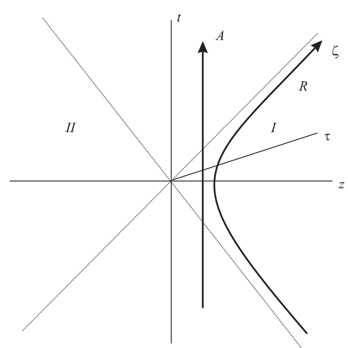

Figure 1: Rindler spacetime diagram: A uniformly accelerated observer Rob (R) moves on a hyperbola with acceleration a in region I and is causally disconnected from region II.

In this paper we work out the effect of decoherence on the entanglement of Dirac field in a noninertial system. Alsing *et al* [7] have shown that the entanglement between two modes of a free Dirac field is degraded by the Unruh effect and asymptotically reaches a nonvanishing minimum value in the infinite acceleration. We investigate that how the loss of entanglement through Unruh effect is influenced in the presence of decoherence by using a phase flip, a phase damping and a depolarizing channel in the Kraus operators formalism. The effect of amplitude damping channel on Dirac field in a noninertial system is recently studied by Wang and Jing [20]. We consider two observers, Alice and Rob, that share a maximally entangled initial state of two qubits at a point in flat Minkowski spacetime. Then Rob moves with a uniform acceleration and Alice stays stationary. To achieve our goal, we consider two cases. In one instance we allow only Rob's qubit to interact with a noisy environment and in the second instance both qubits of the two observers interact with a noisy

environment. Let the two modes of Minkowski spacetime that correspond to Alice and Rob are, respectively, given by $|n\rangle_A$ and $|n\rangle_R$. Moreover, we assume that the observers are equipped with detectors that are sensitive only to their respective modes and share the following maximally entangled initial state

$$|\psi\rangle_{A,R} = \frac{1}{\sqrt{2}} \left(|00\rangle_{A,R} + |11\rangle_{A,R}\right), \tag{1}$$

where the first entry in each ket corresponds to Alice and the second entry corresponds to Rob. From the accelerated Rob's frame, the Minkowski vacuum state is found to be a two-mode squeezed state [7],

$$|0\rangle_M = \cos r|0\rangle_I|0\rangle_{II} + \sin r|1\rangle_I|1\rangle_{II}, \tag{2}$$

where $\cos r = \left(e^{-2\pi\omega c/a} + 1\right)^{-1/2}$. The constant ω, c and a, in the exponential stand, respectively, for Dirac particle's frequency, light's speed in vacuum and Rob's acceleration. In Eq. (2) the subscripts I and II of the kets represent the Rindler modes in region I and II, respectively, in the Rindler spacetime diagram (see Fig. (1)). The excited state in Minkowski spacetime is related to Rindler modes as follow [7]

$$|1\rangle_M = |1\rangle_I|0\rangle_{II}. \tag{3}$$

In terms of Minkowski modes for Alice and Rindler modes for Rob, the maximally entangled initial state of Eq. (1) by using Eqs. (2) and (3) becomes

$$|\psi\rangle_{A,I,II} = \frac{1}{\sqrt{2}} \left(\cos r|0\rangle_A|0\rangle_I|0\rangle_{II} + \sin r|0\rangle_A|1\rangle_I|1\rangle_{II} + |1\rangle_A|1\rangle_I|0\rangle_{II}\right). \tag{4}$$

Since Rob is causally disconnected from region II, we must take trace over all the modes in region II. This leaves the following mixed density matrix between Alice and Rob, that is,

$$\begin{aligned} \rho_{A,I} \quad &= \quad \frac{1}{2}[\cos^2 r|00\rangle_{A,I}\langle 00| + \cos r(|00\rangle_{A,I}\langle 11| + |11\rangle_{A,I}\langle 00|) \\ &\quad\; \sin^2 r|01\rangle_{A,I}\langle 01| + |11\rangle_{A,I}\langle 11|]. \end{aligned} \tag{5}$$

[htb]

2 single qubit in a noisy environment

In this section we consider that only the Rob's qubit is coupled to a noisy environment. The final density matrix of the system in the Kraus operators representation becomes

$$\rho_f = \sum_i \left(\sigma_o \otimes E_i\right) \rho_{A,I} \left(\sigma_o \otimes E_i^\dagger\right), \tag{6}$$

Table 1: A single qubit Kraus operators for phase flip channel, phase damping channel and depolarizing channel.

phase flip channel	$E_o = \sqrt{1-p}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad E_1 = \sqrt{p}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
phase damping channel	$E_o = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \qquad E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$
depolarizing channel	$E_o = \sqrt{1-p}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad E_1 = \sqrt{p/3}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$ $E_2 = \sqrt{p/3}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad E_3 = \sqrt{p/3}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

where $\rho_{A,I}$ is the initial density matrix of the system given by Eq. (5), σ_o is the single qubit identity matrix and E_i are a single qubit Kraus operators of the channel under consideration. The Kraus operators of the channels we use are given in Table 1. The spin-flip matrix of the final density matrix of Eq. (6) is defined as $\tilde{\rho}_f = (\sigma_2 \otimes \sigma_2)\, \rho_f\, (\sigma_2 \otimes \sigma_2)$, where σ_2 is the Pauli matrix. The degree of entanglement in the two qubits mixed state in a noisy environment can be quantified conveniently by concurrence C, which is given as [21, 22]

$$C = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\} \qquad \lambda_i \geq \lambda_{i+1} \geq 0, \tag{7}$$

where λ_i are the eigenvalues of the matrix $\rho_f \tilde{\rho}_f$. The eigenvalues under the action of phase-flip channel becomes

$$\begin{aligned} \lambda_1^{\mathrm{PF}} &= (1 - 2p + p^2)\cos^2 r, \\ \lambda_2^{\mathrm{PF}} &= p^2 \cos^2 r, \\ \lambda_3^{\mathrm{PF}} &= \lambda_4^{\mathrm{PF}} = 0, \end{aligned} \tag{8}$$

where the superscript PF corresponds to phase flip channel. Similarly, the eigenvalues under the action of phase damping and depolarizing channels are, respectively, given by

$$\begin{aligned} \lambda_{1,2}^{\mathrm{PD}} &= \frac{1}{4}(2 - p \pm 2\sqrt{1-p})\cos^2 r, \\ \lambda_3^{\mathrm{PD}} &= \lambda_4^{\mathrm{PD}} = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} \lambda_1^{\mathrm{DP}} &= (-1 + p)^2 \cos^2 r, \\ \lambda_2^{\mathrm{DP}} &= \lambda_3^{\mathrm{DP}} = \lambda_4^{\mathrm{DP}} = \frac{1}{9}p^2 \cos^2 r, \end{aligned} \tag{10}$$

where the superscripts PD and DP stand for phase damping and depolarizing channels, respectively. In all these equations $p \in [0, 1]$ is the decoherence parameter. The upper and lower values of p correspond to undecohered and

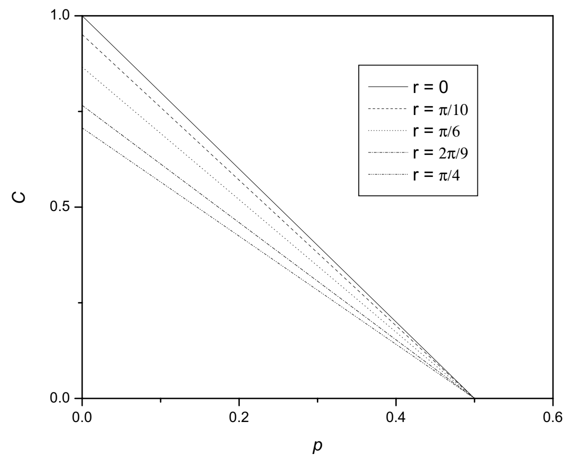

Figure 2: The concurrence C under the action of phase flip channel is plotted against decoherence parameter p for the case when only Rob's qubit is coupled to a noisy environment.

fully decohered case of the channels, respectively. The concurrence under the action of every channel reduces to the result of Ref. [7] when the decoherence parameter $p = 0$.

To see how the concurrence and hence the entanglement is influenced by decoherence parameter p in the presence of Unruh effect, we plot the concurrence for each channel against p for various values of r. In Fig. (2), the concurrence under the action of phase flip channel is plotted against p. The figure shows that for smaller values of p, the entanglement is strongly acceleration dependent, such that for large values of Rob's acceleration (the value of r) it gets weakened. However, as p increases the dependence of entanglement on acceleration decreases and the increasing value of p causes a rapid loss of entanglement. The entanglement sudden death happens irrespective of the acceleration of Rob's frame for a 50% decoherence. Fig. (3) shows the effect of decoherence on the concurrence under the action of phase damping channel. In this case, the degradation of entanglement due to decoherence is smaller as compare to the the degradation in the case of phase flip. The entanglement vanishes for all values of acceleration only when the channel is fully decohered. The concurrence under the action of the depolarizing channel is exactly equal to the one for phase flip channel. Hence it influences the entanglement in a way exactly

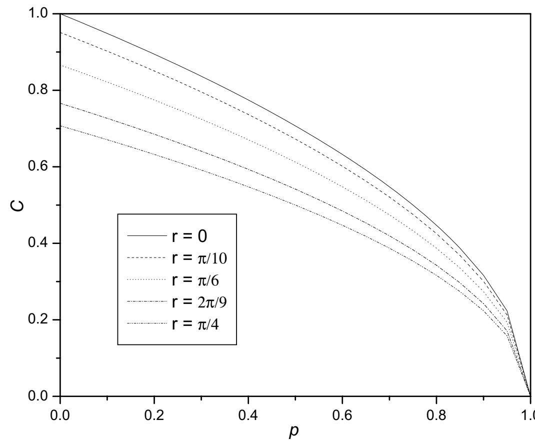

Figure 3: The concurrence C under the action of phase damping channel is plotted against decoherence parameter p for the case when only Rob's qubit is coupled to a noisy environment.

similar to the phase flip channel as shown in Fig. (2).

3 Both qubits in a noisy environment

In this section we consider that both Alice's and Rob's qubits are influenced simultaneously by a noisy environment. The final density matrix in this case can be written in the Kraus operators formalism as follows

$$\rho_f = \sum_k E_k \rho_{A,I} E_k^\dagger, \tag{11}$$

where $\rho_{A,I}$ is given by Eq. (5) and E_k are the Kraus operators for a two qubit system, satisfying the completeness relation $\sum_k E_k E_k = I$ and are constructed from a single qubit Kraus operators of a channel by taking tensor product of all the possible combinations in the following way

$$E_k = \sum_{i,j} E_i \otimes E_j, \tag{12}$$

where $E_{i,j}$ are the single qubit Kraus operators of a channel given in Table 1. We consider that both Alice's and Bob's qubits are influenced by the same environment, that is, the decoherence parameter p for both qubits is same. Proceeding in a similar way like the case of single qubit coupled to the environment, the eigenvalues of the matrix $\rho_f \tilde{\rho}_f$ under the action of phase flip channel become

$$
\begin{aligned}
\lambda_1^{\mathrm{PF}} &= (1+2(-1+p)p)^2 \cos^2 r, \\
\lambda_2^{\mathrm{PF}} &= 4(-1+p)^2 p^2 \cos^2 r, \\
\lambda_3^{\mathrm{PF}} &= \lambda_4^{\mathrm{PF}} = 0,
\end{aligned} \tag{13}
$$

Likewise the eigenvalues for phase damping and depolarizing channels, respectively, becomes

$$
\begin{aligned}
\lambda_1^{\mathrm{PD}} &= \frac{1}{4}(-2+p)^2 \cos^2 r, \\
\lambda_2^{\mathrm{PD}} &= \frac{1}{4} p^2 \cos^2 r, \\
\lambda_3^{\mathrm{PD}} &= \lambda_4^{\mathrm{PD}} = 0,
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\lambda_{1,3}^{\mathrm{DP}} &= \frac{1}{1296}[324 + p(-3+2p)(387 + 152p(-3+2p)) \\
&\quad + 4(3-4p)^2(9+5p(-3+2p)) \cos 2r \\
&\quad + (3-4p)^2 p(-3+2p) \cos 4r \pm 4(3-4p)^2 \cos r \\
&\quad \times \{3(54 + p(-3+2p)(33 + 8p(-3+2p))) \\
&\quad + (3-4p)^2(2(9-6p+4p^2) \cos 2r + p(-3+2p) \cos 4r)\}^{1/2}], \\
\lambda_2^{\mathrm{DP}} &= \lambda_4^{\mathrm{DP}} = \frac{1}{648} p(-3+2p)(-9+4p \\
&\quad + (-3+4p) \cos 2r)(3+4p+(-3+4p) \cos 2r),
\end{aligned} \tag{15}
$$

The "$\pm$" sign in Eq. (15), correspond to the eigenvalues λ_1, and λ_3 respectively. It is necessary to point out here that the concurrence under the action of each channel reduces to the result of Ref. [7] when we set the decoherence parameter $p = 0$.

To see how the entanglement behaves when both the qubits are coupled to the noisy environment, we plot the concurrence against the decoherence parameter p for different values of r under the action of each channel separately. Fig. (4) shows the dependence of concurrence on decoherence parameter p under the action of phase flip channel. The dependence of entanglement on acceleration of Rob's frame is obvious in the region of lower values of p. However, this dependence diminishes as p increases and a rapid decrease in the degree of entanglement develops. At a 50% decoherence level, the entanglement sudden death occurs irrespective of Rob's acceleration. It's interesting to see that

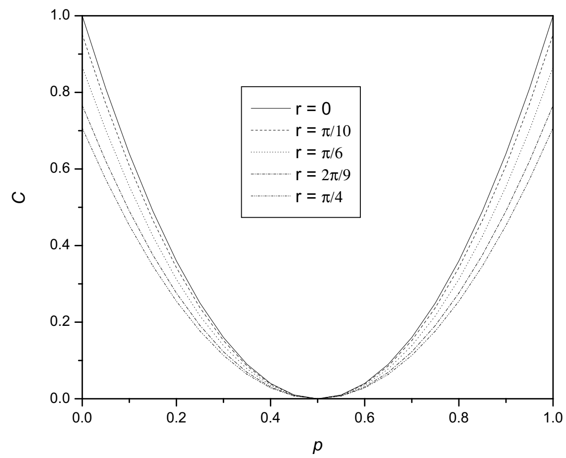

Figure 4: The concurrence C under the action of phase flip channel is plotted against decoherence parameter p for the case when both qubits are coupled to a noisy environment.

beyond this point onward, the entanglement regrows as p increases. The dependence of entanglement on acceleration of the Rob's frame reemerges and the entanglement reaches to the corresponding undecohered maximum value for a fully decohered case. The concurrence varies as a parabolic function of decoherence parameter p with its vertex at $p = 0.5$. The dependence of entanglement on p under the action of phase damping channel is shown in Fig. (5). In this case the entanglement decreases linearly as p increases and the dependence on acceleration diminishes. Whatever the acceleration of Rob's frame may be, the entanglement sudden death occurs when the channel is fully decohered. The influence of depolarizing channel on the entanglement is shown in Fig. (6). Unlike the other two channels, the depolarizing channel does not diminish the effect of acceleration on the entanglement as the p increases. However a rapid decrease in entanglement appears which leads to entanglement sudden death at different values of decoherence parameter for different acceleration of Rob's frame. The larger the acceleration the earlier the entanglement sudden death occurs.

If we compare the single qubit and the both qubits decohering situations, it becomes obvious that the entanglement loss is rapid when both the qubits are coupled to the noisy environment. For example, in the case of bit flip channel the concurrence behaves as a linear function of p for single qubit decohering

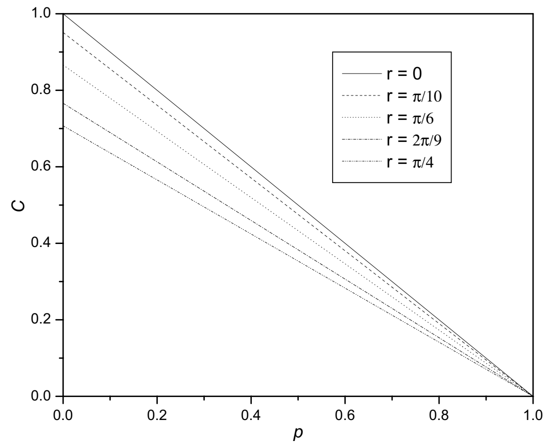

Figure 5: The concurrence C under the action of phase damping channel is plotted against decoherence parameter p for the case when both qubits are coupled to a noisy environment.

case whereas in the case of both qubits decohering case it varies as a parabolic function. Nevertheless, the sudden death happens at the same value of p, irrespective of the acceleration, for both cases under the action of bit flip and phase damping channels. For depolarizing channel, however, this is not true.

4 Conclusion

In conclusion, we have investigated that the entanglement in Dirac fields is strongly dependent on coupling with a noisy environment. This result is contrary to the case of an isolated system in which the entanglement of Dirac fields survives even in the limit of infinite acceleration of Rob's frame. In the presence of decoherence, the entanglement rapidly decreases and entanglement sudden death occurs even for zero acceleration. Under the action of phase flip channel, the entanglement can regrow when both qubits are coupled to a noisy environment in the limit of large values of decoherence parameter. The entanglement disappears, irrespective of the acceleration, under the action of phase damping channel only when the channel is fully decohered both for single qubit and the two qubits decohering cases. However, under the action of depolarizing channel

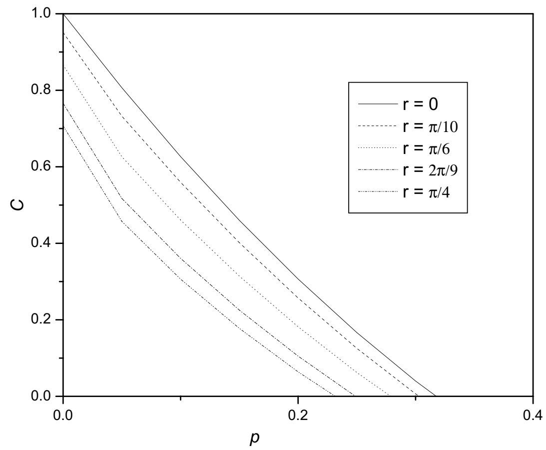

Figure 6: The concurrence C under the action of depolarizing channel is plotted against decoherence parameter p for the case when both qubits are coupled to a noisy environment.

an early sudden death occurs for larger acceleration when both qubits are coupled to the environment. In summary, the entanglement generated by Unruh effect in noninertial frame is strongly influenced by decoherence.

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